MATA2020

MULTIVARIATE APPROXIMATION: THEORY AND APPLICATIONS

Perugia, January 16-18 2020

BOOK OF ABSTRACTS

PLENARY TALKS

Quasiinterpolation and applications using radial basis functions

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In the choice of the approximation of real valued multivariable functions, continuous say, the choice of the linear space from which we approximate the functions is presumably the most important part of the work. In many instances, for radial basis functions, the choice of approximation methods is interpolation, and the spaces are spanned by translates of a radially symmetric kernel function

$$\varphi(\|\cdot-\xi\|), \qquad \xi\in\Xi,$$

where Ξ is a set of (distinct) centres in n-dimensional real space, the vector norm is Euclidean and φ is the radial basis function. For the latter, multiquadrics is one of the most often selected, due to its versatility using a smoothing constant $c\geq 0$ in

$$\varphi(r)=\sqrt{r^2+c^2}, \qquad r\geq 0.$$

In this talk we present new results on theory and applications of quasi-interpolation as compared to interpolation. That is, we form small, usually finite linear combinations of shifts of the above radial basis function and its translates to precondition the basis, to make it local (usually, polynomially decaying) and exact on some low order polynomials.

The theory addresses mainly obtainable decay and convergence properties in L^p (joint work with Feng Dai of the University of Alberta at Edmonton), the application is interested in the solution of partial differential equations (usually, elliptic) from radial basis function spaces. The latter results come from joint work with Joaquin Jodar (University of Jaen) and Miguel Rodriguez (University of Granada).

The obtained work provides both existence theorems and convergence results of the approximations using radial basis functions.

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Sum Rules in Hermite Subdivision

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Hermite subdivision is characterized by the intuition that the vactor valued data to be iterated on consists of value and consecutive derivatives of a function. This leads to a different notion of convergence and also different concepts of polynomial preservation. The talk surveys the underlying structure, in particular the algebra of factorizations, how to obtain extra smoothness and polynomial overreproduction as well as convergence results.

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A new look at approximation problems

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We discuss new results on approximation of functions in Banach function spaces.

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Approximation by deep CNNs in deep learning

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Deep learning has been widely applied and brought breakthroughs in speech recognition, computer vision, natural langaue processing, and many other domains. The involved deep neural network architectures and computational issues have been well studied in machine learning. But there lacks a theoretical foundation for understanding the approximation or generalization ability of deep learning models with network architectures such as deep convolutional neural networks (CNNs) with convolutional structures. The convolutional architecture gives essential differences between the deep CNNs and fully-connected deep neural networks, and the classical approximation theory for fully-connected networks developed around 30 years ago does not apply. This talk describes an approximation theory of deep CNNs. In particular, we show the universality of a deep CNN, meaning that it can be used to approximate any continuous function to an arbitrary accuracy when the depth of the neural network is large enough. Rates of approximation are provided. Our quantitative estimate, given tightly in terms of the number of free parameters to be computed, verifies the efficiency of deep CNNs in dealing with large dimensional data.

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TALKS

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A symbol-related geometric mean of Toeplitz matrices

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Every integrable real-valued function f defined on $I = [0, 2\pi]$ generates, for all n, a real $n \times n$ Toeplitz matrix $T_n(f)$,

$$T_n(f) = \begin{bmatrix} t_0 & t_1 & t_2 & \dots & t_{n-1} \\ t_{-1} & t_0 & t_1 & & \\ t_{-2} & t_{-1} & t_0 & & \vdots \\ \vdots & & & \ddots & \\ t_{-(n-1)} & t_{-(n-2)} & & \dots & t_0 \end{bmatrix}$$

where $t_k = \frac{1}{2\pi}\int_0^{2\pi} f(x)e^{-ikx}dx$ and vice versa if we have a real $n \times n$ Toeplitz matrix T we can obtain a truncated series $\hat{f}(x) = \sum_{k=-(n-1)}^{n-1} t_k e^{ikx}$. In addition, if the matrix T is symmetric, then $t_{-k} = t_k$, thus $\hat{f}$ is a real cosine trigonometric polynomial of degree $n-1$. The function f is frequently called the "symbol" of the matrix T.

In signal processing applications is usual to perform averaging operations involving autocorrelation matrices that have the structure of positive definite (PD) Toeplitz matrices.

In order to define a new mean of PD Toeplitz matrices satisfying all the expected geometrical and structural properties (for example, preserving Toeplitz structure of the matrices), we introduce a parametrization of the PD Toeplitz matrices based on the functional interpretation of Toeplitz matrices. In fact a $n \times n$ PD Toeplitz matrix T can be parameterized by the entries of a $n-$dimensional vector that are the values of the Cesaro sum of order $n-1$ of the symbol of T, evaluated at n different points. Because some opportune choices of these n nodes involve fast trigonometric transforms (discrete cosine transform, in particular), the parametrization and the related mean can be easily computed.

Furthermore, this approach gives us the opportunity to define a geometric-like mean asymptotically well behaving: in fact the symbol of the mean converges to the mean of the symbols as the dimension of the Toeplitz matrices grows.

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Convergence in φ-variation and Rate of Approximation in N-dimension with the Help of Summability Method

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In this work, our aim is to get more general convergence results then the one in [1]. In this approximation, we will consider the convergence of nonlinear integral operators in N-dimension using functions of bounded φ-variation in Tonelli's sense [6, 7]. To get more general estimations, we will use Bell type summability method [4, 5], which consists Cesàro summability, order summability and almost convergence. We will also evaluate the rate of approximation. And then we will give an application of our work to show why we need this generalization. Some other applications of summability method to nonlinear integral operators can be easily found in the following papers [2, 3].

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Inverting Laplace Transform by a *performing* RBF-based fitting model

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Inverting the Laplace Transform starting from discrete data is very frequent in different chemical-physical applications, e.g. those ones based on Nuclear Magnetic Resonance (NMR) techniques. In order to use the numerical scheme to invert a real data set, several algorithms require the evaluation of the Laplace Transform (LT) in some fixed points depending on the inversion method parameters [2,3]. The main idea of this paper is firstly to define a novel fitting model, RBF-based [1], taking into account the analytical properties of the LT, to approximate the data set behaviour; then the approach is promising in using the fitting model in several inversion algorithms that act as black-boxes to invert the discrete data.

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Critical point theory and its applications to difference equations

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The aim of this talk is to discuss some existence and multiplicity results for nonlinear difference equations obtained by using critical point theory in finite dimensional Banach spaces.

More precisely, a survey on some results contained in the references cited below is presented.

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On some generalizations of Bernstein-Durrmeyer operators on hypercubes

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In this talk, which is based on joint works with F. Altomare (cf. [1]) and V. Leonessa (cf. [1, 5]), we describe two generalizations of Bernstein-Durrmeyer operators acting on the d-dimensional hypercube Q_d of $\mathbf{R}^d$, $d \geq 1$.

In particular, we investigate (cf. [1]) a class of operators that generalize the Bernstein-Durrmeyer operators with Jacobi weights on $[0, 1]$ (cf. [4, 6]). Among the motivations to undertake the study of such operators, there is the fact that they are strictly connected with certain degenerate elliptic second-order differential operators, often referred to as Fleming-Viot operators. Fleming-Viot type operators appear in several mathematical models arising from population dynamics, economics and other fields. By making mainly use of approximation theory techniques, it is possible to show that the Fleming-Viot operators (pre)-generate positive semigroups in the space of all continuous functions and in weighted L^p-spaces on Q_d. Moreover, those semigroups may be approximated by means of suitable iterates of the Bernstein-Durrmeyer operators. As a consequence, some spatial regularity properties and the asymptotic behaviour of the semigroups can be inferred.

Inspired by [2, 3], we also present (cf. [5]) a further generalization of the operators in [1]; more precisely, we construct Bernstein-Durrmeyer type operators defined by means of an arbitrary Borel measure μ on Q_d. Some approximation properties of this class of operators, both in the space of all continuous functions and in L^p-spaces with respect to μ, are discussed, together with an asymptotic formula.

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An Adaptive Scheme based on Two Stages for Solving Elliptic PDEs through RBF Collocation Methods

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In this work we present a new adaptive algorithm for solving elliptic partial differential equations (PDEs) via a radial basis function (RBF) collocation method. Our adaptive scheme is based on two stages, which are firstly characterized by the use of a leave-one-out cross validation technique [3], and then on a residual subsampling method [2]. Both phases are meshless and depend on different error indicators and refinement strategies. The combination of these computational approaches enables us to detect the areas that need to be refined, also including the chance to further add or remove adaptively any points. The resulting algorithm is flexible and effective by means of a good interaction between error indicators and refinement procedures. Our study is supported by numerical experiments that illustrate the algorithm performance.

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Two Problems Related To Iterative Sequences and Fixed Points

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Let H be a Hilbert space and let C be a closed, convex and nonempty subset of H. If $T : C \to H$ is a non-self and non-expansive mapping, we can define a map $h : C \to \mathbb{R}$ by $h(x) := \inf\{\lambda \geq 0 : \lambda x + (1 - \lambda)Tx \in C\}$. Then, for a fixed $x_0 \in C$ and for $\alpha_0 := \max\{1/2, h(x_0)\}$, we define the Krasnoselskii-Mann algorithm $x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Tx_n$, where $\alpha_{n+1} = \max\{\alpha_n, h(x_{n+1})\}$. We will prove both weak and strong convergence results when C is a strictly convex set and T is an inward mapping.

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Sampling and stable recovery of planar regions with algebraic boundaries

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We present a new method for the stable reconstruction of a class of binary images from sparse measurements. The images we consider are characteristic functions of algebraic domains, that is, domains defined as zero loci of bivariate polynomials, and we assume to know only a finite set of samples for each image. The solution to such a problem can be set up in terms of linear equations associated to a set of image moments. However, the sensitivity of the moments to noise makes the numerical solution highly unstable. To derive a robust image recovery algorithm, we represent algebraic polynomials and the corresponding image moments in terms of bivariate Bernstein polynomials and apply polynomial-reproducing, refinable sampling kernels. This approach is robust to noise, computationally fast and simple to implement. We illustrate the performance of our reconstruction algorithm from noisy samples through extensive numerical experiments. Our code is released open source and freely available.

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Approximations of fuzzy numbers by severel types of approximation operators

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The aim of this talk is to present some of the main results concerning the approximation of a special class of functions. Namely, upper semicontinuous functions that are quasi-concave with compact support and with maximum value equal to 1. These functions are known in the literature as fuzzy numbers. We will discuss their approximations by Bernstein operators and also by nonlinear Bernstein operators of max-product kind (see [1]). As these nonlinear operators possess a very strong localization property (see [2]), besides the uniform convergence with rates sometimes even better than the linear counterparts, they preserve very well the shape of a fuzzy number. Most notably, they preserve the support, the quasi-concavity and also converge to the modal sets of the approximated fuzzy number. Another approximation method is based on on the so called inverse fuzzy transform which is given in a generalized form in [4]. These operators converge with a Jackson rate of uniform convergence and also preserve the quasi-concavity.

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Lower semi-frames and sequences of integer translates

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A *frame* of a Hilbert space $(\mathcal{H}, \langle\cdot,\cdot\rangle, \|\cdot\|)$ is a sequence of vectors $\{f_n\}_{n\in I} \subset \mathcal{H}$ indexed by a countable set I such that

$$A\|f\|^2 \leq \sum_{n\in I} |\langle f, f_n\rangle|^2 \leq B\|f\|^2, \qquad \forall f \in \mathcal{H} \tag{1}$$

for some $A, B > 0$. Frames have been widely studied since they provide *reconstruction formulas*

$$f = \sum_{n\in I}\langle f, f_n\rangle g_n = \sum_{n\in I}\langle f, g_n\rangle f_n, \qquad \forall f \in \mathcal{H} \tag{2}$$

with $\{f_n\}_{n\in I}$ and $\{g_n\}_{n\in I}$ frames of $\mathcal{H}$.

In this talk we will discuss the reconstruction formulas (at least for a subspace of $\mathcal{H}$) obtained starting with a *lower semi-frame* $\{f_n\}_{n\in I}$, i.e. a sequence satisfying the first inequality in (1). As concrete examples we will consider *sequences of translates* $\{\phi_n\}_{n\in\mathbb{Z}}$ of a function $\phi \in L^2(\mathbb{R})$, i.e. $\phi_n(x) := \phi(x-nb)$ for some $b > 0$ and every $n \in \mathbb{Z}, x \in \mathbb{R}$. These sequences are the building blocks of more complex structured systems like Gabor/wavelet sequences. Moreover, reconstruction formulas like (2) involving $\{\phi_n\}_{n\in\mathbb{Z}}$ are connected to Shannon-type sampling results.

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Numerical solution of Cauchy singular integral equations with additional fixed singularities of Mellin convolution type

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In this talk we propose a quadrature method for the numerical solution of Cauchy singular integral equations with additional terms of Mellin convolution type defined as follows

$$(D+K+H)u(\tau)=g(\tau), \quad |\tau|<1, \tag{1}$$

where

$$(Du)(\tau)=a\,u(\tau)+\frac{b}{\pi}\int_{-1}^{1}\frac{u(t)}{t-\tau}dt,$$

$$(Ku)(\tau)=\int_{-1}^{1}k(t,\tau)u(t)dt,$$

and

$$(Hu)(\tau)=\int_{-1}^{1}h(t,\tau)u(t)dt=g(\tau),$$

with $h(t,\tau)$ and $g(\tau)$ sufficiently smooth functions, $k(t,\tau)$ the Mellin kernel, a and b given real constants such that $a^2+b^2=1$ and $u(\tau)$ the unknown solution.

Since several mathematical problems in physics and engineering can be reduced to the solution of integral equations of the form (1), the development of numerical methods for approximating their solution has been receiving an increasing interest in recent years. In particular, discretization schemes based on polynomial approximation have been considered in [1,2,3], mainly in the case where $k(t,\tau)$ is a special Mellin kernel.

We approximate the unknown function u by a weighted polynomial that is the solution of a finite dimensional equation obtained by discretizing the integral operators by a Gauss-Jacobi quadrature rule. In particular, a slight modification of the latter formula is performed when it is applied to the Mellin integral operator, in order to achieve stability and convergence results. Moreover, we pay a special attention to the study of the conditioning of the involved linear systems, proving that the sequence of their condition numbers converge to the condition number of the operator $D+K+H$. The efficiency of the proposed method is shown through some numerical tests.

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Multinode rational operators for scattered data interpolation: recent advances and future perspectives

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In 1968 D. Shepard [1] introduces an approximation method for the interpolation of scattered data which consists in a weighted average of functional values at the data sites. The method is easy to implement (indeed it is the fastest method for the interpolation of scattered data [2]) but it reproduces exactly only constant polynomials and has flat spots in the neighbourhood of all data points. In 1983 F. Little [3] considers weighted average of local linear interpolants based on triples of data sites and takes as basis functions the normalization of the product of inverse distances from the points of the triples. This method overcomes the drawbacks of the Shepard method and, at the same time, maintains its features of simplicity of implementation and speed [4,5].

As Little suggests, his method can be generalized to higher dimensions and to sets of more than three points. In this talk we will discuss about some generalization and improvement of the triangular Shepard method and set some future direction of research.

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Interpolation of Hermite-type data via scalar subdivision schemes

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Interpolation of Hermite-type data can be achieved by different techniques. A well-established approach to generate smooth curves interpolating a sequence of data points under tangent and curvature constraints consists in looking for either the B-spline representation [1,2,4,5] or the matrix weighted NURBS representation [6] of the sought curve, and in computing either its control points or its weight matrices using the input data. An alternative approach that can be efficiently used to interpolate a sequence of data points and associated derivative vectors, consists in the application of interpolatory vector subdivision schemes of Hermite type [3]. The goal of our work is to achieve interpolation of Hermite-type data (i.e. of points and associated prescribed derivatives) by applying a scalar subdivision scheme to a suitably defined starting polygon obtained from the given sequence of vector valued data. Our approach is extremely flexible since it can be used with both primal and dual scalar subdivision schemes (that include the odd/even degree B-spline representation as a special case), it can handle derivative information of any order and generate planar as well as spatial curves. Several comparative examples are illustrated to point out the effectiveness, the generality and the versatility of the proposed approach. Results obtained with alternative more restrictive approaches can be recovered as special instances of our construction.

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A numerical method for the generalized Love integral equation

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In this talk we present a Nyström type method for the numerical solution of the following generalized Love integral equation

$$f(y) - \frac{1}{\pi}\int_{-1}^{1} \frac{\omega^{-1}}{(x-y)^2+\omega^{-2}} f(x)w(x)dx = g(y), \qquad |y| < 1,$$

where $w(x) = (1-x)^{\alpha}(1+x)^{\beta}$, $\alpha, \beta > -1$, f is the unknown function, g is a known right-hand side, and $0 < \omega \in \mathbb{R}$.

Such equation, which presents a "nearly singular kernel", includes the well-known Love integral equation [1,2] (in the case when $g \equiv w = 1$) and, at the same time, the presence of the weight w leads to the case when the unknown function has algebraic singularities at the endpoints of $[-1, 1]$.

The method is based on a mixed quadrature formula. This is a polynomial product rule whose coefficients are approximated by using a dilation technique. The convergence and the stability of the method is proved in suitable weighted spaces and numerical tests show the accuracy of the method also in the case when the parameter ω is large.

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New smoothness and monotonicity properties of (strictly) positive definite isotropic functions on Hilbert spheres

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In the last five years, there has been a tremendous number of publications stating new results on positive definite functions on spheres. They are for example applied in geostatistics and physiology and are also of importance in statistics where they occur as correlation functions of homogeneous random fields on spheres. Results of Schoenberg and Menegatto [1,2] state:

A function ϕ is positive definite on spheres of arbitrary dimension if and only if it has the form

$$\phi(\theta) = \sum_{m=0}^{\infty} a_m (\cos(\theta))^m, \quad \theta \in [0, \pi],$$

where $a_m \geq 0$, for all $m \in \mathbb{N}_0$, and $\sum_{m=0}^{\infty} a_m < \infty$. It is strictly positive if infinitely of the coefficients a_m are positive for even and odd values of m.

The sequence $\{a_m\}_{m \in \mathbb{N}}$ is referred to as ∞-Schoenberg sequence. In the talk, we will present results that allow to deduce properties of the Schoenberg sequence from smoothness or monotonicity properties of the function (partly published in [3]). For example, the smoothness of the isotropic function in zero allows to deduce certain summability properties of the coefficients and monotonicity properties can be used to prove the strict positive definiteness of an isotropic function. The results extend to conditionally positive definite functions and general spheres in some special cases.

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On Urysohn type integral form of Generalized Sampling Operators

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The aim of this work is to define and study Urysohn type integral form of generalized sampling operators by using the Urysohn type interpolation of the given function f. The basis used in this construction are the Prenter Denstity theorem and Urysohn type operator values instead of the rational sampling values of the function. After that, we investigate some properties of this operators in some function spaces.

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1

On the cosine operator function framework of certain approximation processes in Banach spaces

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Let X be a Banach space, and $[X]$ the Banach algebra of all bounded linear operators $U : X \to X$.

Definition 1. The cosine operator function $C_h \in [X]$ $(h \geq 0)$ is defined by the properties:
(i) $C_0 = I$(identity operator),
(ii) $C_{h_1} \cdot C_{h_2} = \frac{1}{2}(C_{h_1+h_2} + C_{|h_1-h_2|})$,
(iii)$\|C_h f\| \leq T\|f\|$, the constant $T > 0$ is not depending on $h > 0$.

Let $A_\sigma \subset X$ be a dense family of linear subspaces with $A_{\sigma_1} \subset A_{\sigma_2}, 0 < \sigma_1 < \sigma_2$, meaning that for every $f \in X$ there exists a family $\{g_\sigma\}_{\sigma>0} \subset \bigcup\limits_{\sigma>0} A_\sigma$ such that $\lim_{\sigma\to\infty} \|f - g_\sigma\| = 0$. Let $A_\sigma \subset X$ consist of the fixed points of a linear operator $S_\sigma : A_\sigma \to A_\sigma$, i.e. for any $g \in A_\sigma$ we have $S_\sigma g = g$.

Now we may define our approximation operators in subspaces A_σ.

Definition 2. The cosine-type approximation operators $\widetilde{U}_{\sigma,h,\mathbf{a}} : A_\sigma \to X$ are defined by

$$\widetilde{U}_{\sigma,h,\mathbf{a}} g := \sum_{k=0}^{m} a_k C_{kh} \left(S_\sigma g\right), \ h \geq 0,$$

where $\mathbf{a} = (a_0, ..., a_m) \in \mathbb{R}^{m+1}, \ m \geq 1$ and $\sum_{k=0}^{m} a_k = 1$.

By the Bounded Linear Transformation Theorem the operators $\widetilde{U}_{\sigma,h,\mathbf{a}} : A_\sigma \to X$ can be uniquely extended to a bounded linear transformation $U_{\sigma,h,\mathbf{a}} : X \to X$. Typically, the order of approximation by certain operators has been estimated by the modulus of continuity of order $k \in \mathbb{N}$, and by the best approximation, defined, respectively, by

$$\omega_k(f, \delta) := \sup_{0\leq h\leq\delta} \|(C_h - I)^k f\|, \quad E_\sigma(f) := \inf_{g\in A_\sigma} \|f - g\|.$$

In the given above framework we can prove (see [3])

Theorem 1. *Assume* $\sum_{k=0}^{m} a_k = 1$ *is valid. Then for every* $f \in X$ *for the operators* $U_{\sigma,h,\mathbf{a}} : X \to X$ *we have*

$$\|U_{\sigma,h,\mathbf{a}} f - f\| \leq \left(\|U_{\sigma,h,\mathbf{a}}\|_{[X]} + T \sum_{k=0}^{m} |a_k| \right) E_\sigma(f) + \max(T, 1)\omega(f, h) \sum_{l=1}^{m} l^2 \left|a_l\right|.$$

and its improvement via the higher order modulus of continuity $\omega_k(f, h)$. Our theorems can be applied for the generalized Shannon sampling operators [2] and for the trigonometric approximation operators (Fourier series and transforms [1]) as well.

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The Material Point Method for large deformation problems in engineering. A grid-based vs a mesh-free approach

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The Material Point Method (MPM) is an hybrid numerical technique falling under the category of the so-called continuum-based particle methods. These methods have recently been proposed and developed to overcome the limitation of the classical finite element method (FEM) when dealing with large deformation problems, common in many engineering applications. FEM approaches cannot be used when the mesh is severely distorted unless frequent remeshing are performed and the need for alternative techniques is thus evident [1].

Among the possible solutions, the material point method is chosen because it blends all the advantages of both Eulerian and Lagrangian approaches, avoiding the need for frequent remeshing which is not only computationally expensive, but also a source of inaccuracy.

MPM uses a set of Material Points to describe the body under analysis and a background Eulerian grid to solve the governing equation in an updated Lagrangian framework. All the historical variables are stored in the material points, while the mesh information is reset at any time step. MPM can be seen as a finite element method with moving integration points represented by the material points.

In this work we present a classical MPM formulation (grid based) and a mesh-free MPM formulation to solve mass and linear momentum balance equations in a finite strain regime [2]. The mesh free MPM represents the application of the MPM idea to the case in which both the nodes and the material points are considered as Lagrangian. Differently from the grid-based algorithm, the position of the nodes evolves through the whole simulation, so that the nodes preserve their history and can be used to store historical variables. The meshless MPM algorithm represents a very natural generalization of a traditional grid-based one. In this latter case linear elements are used, while in the former we use Moving Least Squares and Local Maximum Entropy approximants [3].

We will present the derivation of the stabilized algebraic solution system for both methods and we will assess the performance of the two techniques for the solution of classical benchmark tests in solid mechanics.

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Kantorovich-type operators on mobile intervals

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In this talk we are concerned with a new sequence of positive linear operators acting on the space of integrable functions on $[0,1]$. Inspired to [1], if $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ are two sequences of real numbers such that $0 \leq a_n < b_n \leq 1$ $(n \geq 1)$, such operators are defined in the following way: for every $n \geq 1$, $f \in L^1([0,1])$, and $0 \leq x \leq 1$,

$$C_n^\tau(f)(x) = \sum_{k=1}^{n} \binom{n}{k} \tau(x)^k (1-\tau(x))^{n-k} \left(\frac{n+1}{b_n - a_n} \int_{\frac{k+a_n}{n+1}}^{\frac{k+b_n}{n+1}} (f \circ \tau^{-1})(t)\, dt \right),$$

where $\tau \in C^\infty([0,1])$ with $\tau(0) = 0$, $\tau(1) = 1$, and $\tau' > 0$ on $[0,1]$. Note that, for $\tau(x) = x$ $(0 \leq x \leq 1)$, $a_n = 0$ and $b_n = 1$ $(n \geq 1)$, the above operators turn into the classical Kantorovich operators. Further, the operators C_n^τ can be viewed as an integral modification with mobile intervals of the Bernstein-type operators B_n^τ introduced and studied in [3].

In particular, we discuss some qualitative properties of operators C_n^τ, in particular we show that they preserve some generalized form of convexity. Moreover, we prove that $(C_n^\tau)_{n\geq 1}$ is an approximation process for spaces of continuous functions, as well as integrable functions, and we evaluate the rate of convergence in both cases in terms of suitable moduli of smoothness. Further, we determine an asymptotic formula which allows us to show that certain iterates of the operators C_n^τ converge, both in $C([0,1])$ and, in some cases, in $L^p([0,1])$, to a limit semigroup.

We end by noting that, under suitable conditions, the new operators perform better than the ones contained in [1]. Finally, as a byproduct, we get an approximation results for the derivatives of the operators B_n^τ.

This is a joint work with Mirella Cappelletti Montano (see [2]).

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Approximating the solutions of differential inclusions driven by measures

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The matter of approximating the solutions of a differential problem driven by a rough measure by solutions of similar problems driven by “smoother" measures is considered under very general assumptions on the multifunction on the right hand side. The key tool in our investigation is the notion of uniformly bounded ε-variations, which mixes the supremum norm with the uniformly bounded variation condition. Several examples to motivate the generality of our outcomes are included.

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Boundary Point Method and the Mann-Dotson Algorithm in Banach Spaces

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Let C be a closed, convex and nonempty subset of a Banach space X. Let $T : C \to X$ be a nonexpansive inward mapping. We consider the boundary point map $h_{C,T} : C \to \mathbb{R}$ depending on C and T defined by $h_{C,T} = \max\{\lambda \in [0,1] : [(1-\lambda)x + \lambda Tx] \in C\}$, for all $x \in C$. Then for a suitable step-by-step construction of the control coefficients by using the function $h_{C,T}$, we show the convergence of the Mann-Dotson algorithm to a fixed point of T. We obtain strong convergence if $\sum_{n\in\mathbb{N}} \alpha_n < \infty$ and weak convergence if $\sum_{n\in\mathbb{N}} \alpha_n = \infty$.

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On the Solution of Fractional Differential Problems by Quasi-Interpolant Operators

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Nowadays, Fractional Calculus is a well-established tool to model a real variety of real-world phenomena, from viscoelasticity to population growth, from anomalous diffusion to wave propagation [1]. The grown popularity of differential problems having derivatives of fractional, i.e. noninteger, order is due to their ability to model nonlocality in space or memory effects in time. Unfortunately, the analytical solution of fractional differential problems is known just in some special cases and it is often expressed as a series expansion [2]. Thus, efficient numerical methods to solve fractional differential problems are very welcomed [3].
In [4] the authors proposed to solve fractional differential problems by a collocation method based on interpolation operators in refinable spaces. In this talk we extend this collocation method to the case of quasi-interpolant operators [5]. We analyze the approximation properties of the method and conduct some numerical tests showing its performance.

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Cheating with the domain: the Fake Nodes approach as an interpolation paradigm

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Mapping with a suitable function the domain points can lead in many circumnstances to a more accurate and stable interpolation in every interpolation scheme used.
Our approach consists in applying a proper map S to both the interpolation nodes $\mathcal{X}$ and evaluation nodes $\mathcal{X}_{ev}$ before the interpolation process and to associate the resulting interpolating values to the (unmapped) evaluation nodes.
This process is equivalent to the interpolation on a mapped basis

$$\{b_1 \circ S, \ldots, b_n \circ S\},$$

being $\{b_1, \ldots, b_n\}$, the 'standard' basis of interpolating functions.
This method can be applied successfully to every interpolation method, for instance polynomials, RBF, splines or rational functions.

In this talk we will show the theoretical background of this approach along with several applications in 1D and 2D interpolation, as the treatment of Runge and Gibbs phenomena.

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On derivative sampling using Kantorovich-type sampling operators

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In [1] and [2] L. Angeloni, D. Costarelli and G. Vinti give a connection between generalized sampling operators and sampling Kantorovich operators in form

$$(K_w^{\chi} f)(x) = (G_w^{\overline{\chi}} F)' \left(x + \frac{1}{2w} \right), \tag{1}$$

where sampling Kantorovich operators

$$(K_w^{\chi} f)(x) := \sum_{k=-\infty}^{\infty} w \left(\int_{k/w}^{(k+1)/w} f(u)\, du \right) \chi(wx - k),$$

and generalized sampling operators

$$(G_w^{\overline{\chi}} F)(x) := \sum_{k=-\infty}^{\infty} F\left(\frac{k}{w} \right) \overline{\chi}(wx - k)$$

with

$$\overline{\chi}(t) := \int_{-1/2}^{1/2} f(t + v)\, dv, \quad F(x) := \int_0^x f(t)\, dt.$$

In this talk we use generalized Kantorovich-type sampling operators, we introduced in [3], and show a connection, similar to (1), between generalized sampling operators with averaged kernels and generalized Kantorovich-type sampling operators. We can use related kernels, introduced in [4], to give some estimates for the order of approximation of derivatives with generalized sampling operators.

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Dual Interpolatory Subdivision

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A new class of univariate stationary interpolatory subdivision schemes of dual type is presented. As opposed to classical their primal counterparts, these new schemes have basic limit function φ which satisfies a refinement relation of the type

$$\varphi(x) \;=\; \sum_{k=1-k^*}^{k^*} a_k\, \varphi(mx-k+1/2),$$

for some arity $m \in \mathbb{N} \setminus \{1,2\}$ and mask $\mathbf{a} = [a_k]_{1-k^*}^{k^*}$, $k^* \in \mathbb{N}$, with an even number of elements. Moreover, the interpolation is not step-wise but is only achieved in the limit. A complete algebraic characterization, which covers every arity, is given in terms of identities related to the trigonometric polynomials

$$A_n(z) \;=\; \frac{1}{m} \sum_{k\in\mathbb{Z}} a_{mk+n}\, z^{mk+n},$$

$$\Phi_n(z) \;=\; \frac{1}{2} \sum_{k\in\mathbb{Z}} \varphi\left(mk+\frac{n}{2}\right)\, z^{2mk+n},$$

with $n \in \mathbb{Z}$, $|z| = 1$. This characterization is based on a necessary condition for φ to have prescribed values at $\mathbb{Z}/2$ (similar to [1]), which a consequence of the Poisson summation formula. A strategy for the construction as well as examples in both the stationary and non-stationary case will be shown.

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POSTERS

Poster Session:

1. Prof. Laura Angeloni, University of Perugia (Italy)
   “Approximation in BV spaces in multidimensional frame by means of sampling type operators”

2. Prof. Marco Cantarini, University of Perugia (Italy)
   “Neural Network operators: Approximation results”

3. Prof. Monica Dessole, University of Padova (Italy)
   “Efficient computation of large-scale Tchakaloff regression designs”

4. Prof. Filomena Di Tommaso, University of Calabria (Italy)
   “On the numerical stability of multinode Shepard operators”

5. Prof. Marta Gatto, University of Padova (Italy)
   “An algorithm for model-based denoising of input-output data”

6. Prof. Francesco Marchetti, University of Padova (Italy)
   “Fusing SVM and Naive Bayes via VSKs”

7. Prof. Donatella Occorsio, University of Basilicata (Italy)
   “Optimal Lebesgue constants in [-1,1]^2”

8. Prof. Emma Perracchione, University of Padova (Italy)
   “RBFs and tensors for scoring wines”

9. Prof. Marco Seracini, University of Perugia (Italy)
   “Multivariate Sampling Kantorovich Operators: a segmentation procedure of the aorta artery from CT images without contrast medium”

10. Prof. Woula Themistoclakis, CNR-IAC “Mauro Picone” (Italy)
    “Filtered polynomial interpolation on the square”

11. Prof. Franco Zivcovich, University of Trento (Italy)
    “Fast and accurate computation of divided differences for analytic functions, with an application to the exponential function”